



## دبیرستان پسرانه غیر دولتی ابتکار علم - دوره دوم

نام و نام خانوادگی: سوال و راهنمای تصحیح کلاس: یازدهم ریاضی موضوع امتحان: ریاضی ۲ نام دبیر: استاد اسماعیل پور

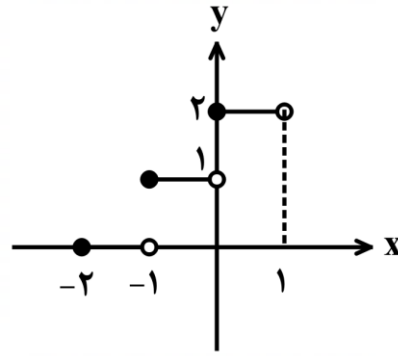
ردیف	راهنمای تصحیح
۱	$x_c = \frac{2+6}{2} = 4$ $y_c = \frac{-2+4}{2} = 1$ $R = \sqrt{(6-4)^2 + (4-1)^2} = \sqrt{4+9} = \sqrt{13}$
۲	$a = \frac{ 6-0-1 }{\sqrt{4+1}} = \frac{5}{\sqrt{5}} = \sqrt{5} \rightarrow S = a^2 = 5$
۳	$\begin{cases} S = 1 + \sqrt{2} + 1 - \sqrt{2} = 2 \\ P = (1 + \sqrt{2})(1 - \sqrt{2}) = -1 \end{cases} \rightarrow x^2 - Sx + P = 0 \rightarrow x^2 - 2x - 1 = 0$
۴	$PQ \parallel BC \rightarrow \begin{cases} \frac{AP}{PB} = \frac{AQ}{QC} \rightarrow \frac{x}{6-x} = \frac{2}{3} \rightarrow x = 4 = AP \\ \frac{AQ}{AC} = \frac{PQ}{BC} \rightarrow \frac{2}{5} = \frac{y}{9} \rightarrow y = \frac{18}{5} = 3.6 = PQ \end{cases}$
۵	$BC^2 = AB^2 + AC^2 \rightarrow BC^2 = 64 + 36 = 100 \rightarrow BC = 10$ $AH \cdot BC = AB \cdot AC \rightarrow AH \times 10 = 8 \times 6 \rightarrow AH = 4.8$
۶	<p>(الف)</p> $D_f = [-2, +\infty), D_g = \mathbb{R} - \{3\}, f(x) = 0 \rightarrow \sqrt{x+2} = 0 \rightarrow x = -2$ $D_{\frac{g}{f}} = D_f \cap D_g - \{x \mid f(x) = 0\} = [-2, 3) \cup (3, +\infty) - \{-2\} = (-2, 3) \cup (3, +\infty)$ <p>(ب)</p> $(3f - 2g)(-1) = 3f(-1) - 2g(-1) = 3(1) - 2\left(\frac{-1}{2}\right) = 4$

$$y = [x] + 2 \quad D_f = [-2, 1)$$

$$-2 \leq x < -1 \rightarrow y = -2 + 2 \rightarrow y = 0$$

$$-1 \leq x < 0 \rightarrow y = -1 + 2 \rightarrow y = 1$$

$$0 \leq x < 1 \rightarrow y = 0 + 2 \rightarrow y = 2$$



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$$L = r \cdot \theta \rightarrow \lambda = 1 \cdot \theta \rightarrow \theta = \frac{4}{5} \text{ rad}$$

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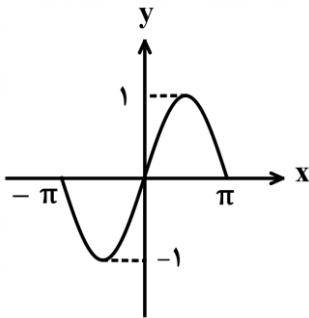
الف)  $\sin\left(\frac{25\pi}{3}\right) = \sin\left(8\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

ب)  $\cos\left(\frac{23\pi}{4}\right) = \cos\left(6\pi - \frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

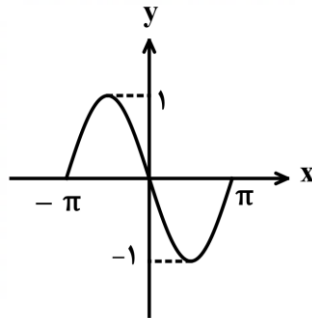
ج)  $\tan(135^\circ) = \tan(\pi - 45^\circ) = -\tan 45^\circ = -1$

د)  $\cot(240^\circ) = \cot(\pi + 60^\circ) = \cot 60^\circ = \frac{\sqrt{3}}{3}$

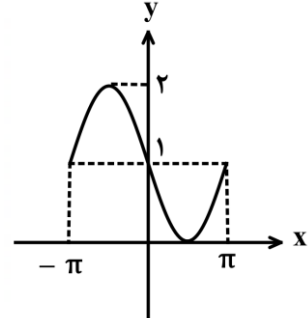
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$$y = \sin x$$



$$y = -\sin x$$



$$y = 1 - \sin x$$

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$$9^{3x-2} = 27^{x+1} \rightarrow 3^{6x-4} = 3^{3x+3} \rightarrow 6x-4 = 3x+3 \rightarrow 3x=7 \rightarrow x=7/3$$

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$$\log_{1/3} 81 - 4 \log_{1/9} \frac{1}{49} + 7 \log_{1/10} \dots 1 = \log_{3^{-1}} 3^4 - 4 \log_{9^{-1}} 7^{-2} + 7 \log_{10^{-1}} 10^{-7} = -4 + 8 - 7 = -3$$

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$$\log_{\Delta}(x+1) + \log_{\Delta}(x-1) = 1 \rightarrow \log_{\Delta}(x+1)(x-1) = 1 \rightarrow (x+1)(x-1) = \Delta \rightarrow x^2 - 1 = \Delta$$

$$\rightarrow x^2 = \Delta + 1 \rightarrow \begin{cases} x = \sqrt{\Delta + 1} \text{ ق ق} \\ x = -\sqrt{\Delta + 1} \text{ غ ق} \end{cases}$$

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$f(x) = \begin{cases} -x + 2 & x > 2 \\ -2 & x = 2 \\ x - 3 & x < 2 \end{cases}$ $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (-x + 2) = 0$ $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x - 3) = -1$ <p><math>\lim_{x \rightarrow 2^+} f(x) \neq \lim_{x \rightarrow 2^-} f(x) \rightarrow</math> پس تابع در <math>x = 2</math> حد ندارد</p>	۱۴
<p>الف) <math>\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - 3x + 2} = \lim_{x \rightarrow 2} \frac{x(x-2)}{(x-2)(x-1)} = \lim_{x \rightarrow 2} \frac{x}{x-1} = 2</math></p> <p>ب) <math>\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\cos^2 x} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{1 - \sin^2 x} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{(1 - \sin x)(1 + \sin x)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{1 + \sin x} = \frac{1}{2}</math></p>	۱۵
<p><math>f(\cdot) = 2, \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (-2x + 2) = 2, \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (x^2 + 2) = 2</math></p> <p>چون <math>f(\cdot) = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)</math> پس تابع در <math>x = 0</math> پیوسته است.</p>	۱۶
<p><math>P(A) = \frac{1}{5}, P(B) = \frac{1}{7}, P(B A) = \frac{1}{4}, P(A \cup B) = ?</math></p> <p><math>P(B A) = \frac{P(B \cap A)}{P(A)} = \frac{1}{4} = \frac{P(B \cap A)}{\frac{1}{5}} \rightarrow P(B \cap A) = \frac{1}{20}</math></p> <p><math>P(A \cup B) = P(A) + P(B) - P(A \cap B) \rightarrow P(A \cup B) = \frac{1}{5} + \frac{1}{7} - \frac{1}{20} \rightarrow P(A \cup B) = \frac{41}{140}</math></p>	۱۷
<p>۹, ۱۱, ۱۲, ۱۳, ۱۵</p> <p><math>\bar{X} = \frac{\sum X_i}{n} = \frac{9+11+12+13+15}{5} = 12</math></p> <p><math>\sigma^2 = \frac{\sum (X_i - \bar{X})^2}{n} = \frac{9+1+0+1+9}{5} = 4</math></p> <p><math>\sigma = \sqrt{\sigma^2} = \sqrt{4} = 2, CV = \frac{\sigma}{\bar{X}} = \frac{2}{12} = \frac{1}{6}</math></p>	۱۸
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